

Determination of the quarkonium–gluonium content of the isoscalar tensor resonances $f_2(1920)$, $f_2(2020)$, $f_2(2240)$, $f_2(2300)$ and of the broad state $f_2(2000)$ based on decay couplings to $\pi^0\pi^0, \eta\eta, \eta\eta'$

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In the reactions $p\bar{p} \rightarrow \pi^0\pi^0, \eta\eta, \eta\eta'$ there are four relatively narrow resonances $f_2(1920)$, $f_2(2020)$, $f_2(2240)$, $f_2(2300)$, and a broad one $f_2(2000)$ in the mass region 1990–2400 MeV. In the framework of quark combinatorics we carry out an analysis of the decay constants for all five resonances. It is shown that the relations for the decay constants corresponding to the broad resonance $f_2(2000) \rightarrow \pi^0\pi^0, \eta\eta, \eta\eta'$ are the same as those corresponding to a glueball. An additional argument in favour of the glueball–nature of $f_2(2000)$ is the fact that $f_2(1920)$, $f_2(2020)$, $f_2(2240)$, $f_2(2300)$ fit well the $q\bar{q}$ trajectories in the (n, M^2) -plane (where n is the radial quantum number), while the broad $f_2(2000)$ resonance turns out to be an unnecessary extra state for these trajectories.

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I. INTRODUCTION

A broad f_2 -resonance was observed in the region of 2000 MeV in various reactions (see the compilation [1] and references therein). The measured masses and widths are:

$M = 2010 \pm 25$ MeV, $\Gamma = 495 \pm 35$ MeV in $p\bar{p} \rightarrow \pi^0\pi^0, \eta\eta, \eta\eta'$ [2],

$M = 1980 \pm 20$ MeV, $\Gamma = 520 \pm 50$ MeV in $pp \rightarrow pp4\pi$ [3],

$M = 2050 \pm 30$ MeV, $\Gamma = 570 \pm 70$ MeV in $\pi^-p \rightarrow \phi\phi n$ [4].

Following these measurements, we denote the broad resonance as $f_2(2000)$.

A recent re-analysis of the $\phi\phi$ spectra [4] in the reaction $\pi^-p \rightarrow \phi\phi n$ [5], and the analysis of the process $\gamma\gamma \rightarrow K_S K_S$ [6] has essentially clarified the status of the $(I = 0, J^{PC} = 2^{++})$ -mesons. This enables us to place them on the (n, M^2) -trajectories, where n is the radial quantum number of the $q\bar{q}$ -state.

In [7] (see also [8–10]) we put the known $q\bar{q}$ -mesons consisting of light quarks ($q = u, d, s$) on the (n, M^2) trajectories. Trajectories for mesons with various quantum numbers turn out to be linear with a good accuracy:

$$M^2 = M_0^2 + (n-1)\mu^2 \quad (1)$$

where $\mu^2 = 1.2 \pm 0.1$ GeV² is a universal slope, and M_0 is the mass of the lowest state with $n = 1$.

On Fig. 1 we show the present status of the (n, M^2) trajectories for the f_2 -mesons (i.e. we use the results given by [4,6]). To avoid confusion, we list here the experimentally observable masses.

In [4] the $\phi\phi$ spectra are re-analysed, taking into account the existence of the broad $f_2(2000)$ resonance. As a result, the masses of three relatively narrow resonances are shifted compared to those given in the compilation PDG [1]:

$$\begin{aligned} f_2(2010)|_{PDG} &\rightarrow f_2(2120) [4] , \\ f_2(2300)|_{PDG} &\rightarrow f_2(2340) [4] , \\ f_2(2340)|_{PDG} &\rightarrow f_2(2410) [4] . \end{aligned} \quad (2)$$

A phase analysis for the reaction $\gamma\gamma \rightarrow K_S K_S$ is carried out in [6]; the characteristics of $f_2(1755)$ are measured. This resonance belongs to the nonet of the first radial excitation, ($n = 2$); it is dominated by a $^3P_2 s\bar{s}$ state. Its partner in the ($n = 2$) nonet is $f_2(1580)$. Neglecting a possible admixture of a glueball component, in [6]

$$\begin{aligned} f_2(1580) &= n\bar{n} \cos \varphi_{n=2} + s\bar{s} \sin \varphi_{n=2} , \\ f_2(1755) &= -n\bar{n} \sin \varphi_{n=2} + s\bar{s} \cos \varphi_{n=2} , \\ \varphi_{n=2} &= -10^\circ \text{ } ^{+5^\circ}_{-10^\circ} \end{aligned} \quad (3)$$

is found. Here $n\bar{n} = (u\bar{u} + s\bar{s})/\sqrt{2}$.

The quark states with $(I = 0, J^{PC} = 2^{++})$ are determined by two flavour components $n\bar{n}$ and $s\bar{s}$ for which two states $^{2S+1}L_J = ^3P_2, ^3F_2$ are possible. Consequently, we have four trajectories on the (n, M^2) plane. Generally speaking, the f_2 -states are mixtures of both the flavour components and the $L = 1, 3$ waves. The real situation is, however, such that the lowest trajectory $[f_2(1275), f_2(1580), f_2(1920), f_2(2240)]$ consists of mesons with dominant $^3P_2 n\bar{n}$ components, while the trajectory $[f_2(1525), f_2(1755), f_2(2120), f_2(2410)]$ contains mesons with predominantly $^3P_2 s\bar{s}$ components, and the F -trajectories are represented by three resonances $[f_2(2020), f_2(2300)]$ and $[f_2(2340)]$ with the corresponding dominant $^3F_2 n\bar{n}$ and $^3F_2 s\bar{s}$ states. Similarly to [10], the broad resonance $f_2(2000)$ is not part of those states placed on the (n, M^2) trajectories. In the region of 2000 MeV we see three resonances, $f_2(1920)$, $f_2(2000)$, $f_2(2020)$, while on the (n, M^2) -trajectories there are only two vacant places. This means that one state is obviously "superfluous" from the point of view of the $q\bar{q}$ -systematics, i.e. it has to be considered as exotics. The large value of the width of $f_2(2000)$ strengthen the suspicion that, indeed, this state is an exotic one.

In [11] it was pointed out that an exotic state has to be broad. Indeed, if an exotic resonance occurs among the usual $q\bar{q}$ -states, they overlap, and their mixing becomes possible owing to the *resonance* (1) \rightarrow *real mesons* \rightarrow *resonance* (2) transitions at large distances. It is due to these transitions that an exotic meson accumulates the widths of its neighbouring resonances. The phenomenon of the accumulation of widths was shown in the scalar sector near 1500 MeV [12,13]. In fact the accumulation of widths was observed much earlier, in overlapping resonances in nuclear physics [14,?,16]. Hence, the large width of $f_2(2000)$ can indicate that this state is an exotic one. Still, to verify that $f_2(2000)$ is of glueball nature, it is necessary to investigate the relations between the coupling constants of $f_2(2000)$ with the meson channels. The coupling constants of the transitions

$$f_2(1920), f_2(2000), f_2(2020), f_2(2240), f_2(2300) \rightarrow \pi\pi^0, \eta\eta, \eta\eta'$$

were obtained in [2,17] from the analysis of the reactions $p\bar{p} \rightarrow \pi^0\pi^0, \eta\eta, \eta\eta'$. To investigate the quarkonium ($q\bar{q}$) and gluonium (gg) contents of $f_2(1920)$, $f_2(2000)$, $f_2(2020)$, $f_2(2240)$, $f_2(2300)$, we make use of these coupling constants in Section 2. It turns out that only the coupling constants corresponding to $f_0(2000) \rightarrow \pi^0\pi^0, \eta\eta, \eta\eta'$ satisfy the relations which characterize the glueball (they are close to those given by a flavour singlet). In the Conclusions we discuss the flavour content of the $q\bar{q}$ states, and compare the properties of a 2^{++} glueball with those of a 0^{++} glueball, situated in the 1500 MeV region.

II. THE DETERMINATION OF THE $Q\bar{Q} - GG$ CONTENT OF ISOSCALAR TENSOR MESONS, OBSERVED IN THE REACTIONS $P\bar{P} \rightarrow \pi^0\pi^0, \eta\eta, \eta\eta'$

On the basis of data given by the analysis of the reactions $p\bar{p} \rightarrow \pi^0\pi^0, \eta\eta, \eta\eta'$ [2], in this section we investigate the quarkonium–gluonium content of $f_2(1920)$, $f_2(2000)$, $f_2(2020)$, $f_2(2240)$, $f_2(2300)$. We do this in terms of the rules of quark combinatorics.

A. The rules of quark combinatorics for decay constants

The isoscalar tensor $q\bar{q}$ mesons, close to the 2^{++} gluonium state and mixing with it, are superpositions of the components $\sqrt{1-W} q\bar{q} + \sqrt{W} gg$. The quarkonium component is, in its turn, a mixture of $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$:

$$q\bar{q} = n\bar{n} \cos \varphi + s\bar{s} \sin \varphi . \quad (4)$$

In terms of the $1/N$ expansion rules [18] the transitions of the $q\bar{q}$ and gg components are not suppressed (the contribution of the quark loop in the gluon ladder is of the order of N_f/N_c , where N_f is the number of light flavours, N_c that of the colours; see, e.g., Subsection 5.4.4 in [8] and references therein). Hence, it is justified to expect relevant admixtures of gluonic components to states which we consider $q\bar{q}$ -mesons (i.e. to those lying on the (n, M^2) trajectories in Fig. 1).

On the other hand, a glueball state has to contain essential $q\bar{q}$ components:

$$\begin{aligned} gg \cos \gamma + (q\bar{q})_{glueball} \sin \gamma , \\ (q\bar{q})_{glueball} = n\bar{n} \cos \varphi_{glueball} + s\bar{s} \sin \varphi_{glueball} . \end{aligned} \quad (5)$$

If the flavour SU(3) symmetry were satisfied, the quarkonium component $(q\bar{q})_{glueball}$ would be a flavour singlet. In reality, the probability of strange quark production in a gluon field is suppressed: $u\bar{u} : d\bar{d} : s\bar{s} = 1 : 1 : \lambda$, where

$\lambda \simeq 0.5 - 0.85$. Hence, $(q\bar{q})_{glueball}$ differs slightly from the flavour singlet: $(q\bar{q})_{glueball} = (u\bar{u} + d\bar{d} + \sqrt{\lambda} s\bar{s})/\sqrt{2+\lambda}$ [19]. The suppression parameter (λ) for the production of a strange quark was estimated both in multiple hadron production processes [20], and in hadronic decay processes [21,22]. According to these estimations, λ is of the order of 0.5–0.85, which leads to

$$\varphi_{glueball} \simeq 26^\circ - 33^\circ. \quad (6)$$

The hadronic decay processes of quarkonium and gluonium states are determined by production processes of new quark-antiquark pairs. In the leading terms of the $1/N$ expansion the vertices of hadronic resonance decays are determined by planar diagrams. Examples of planar diagrams for the decay of a $q\bar{q}$ state and of a glueball into two $q\bar{q}$ mesons are shown in Fig. 2; in the decay process of a $q\bar{q}$ state the gluons produce a new $q\bar{q}$ -pair, while the decay of a glueball leads to the production of two $q\bar{q}$ -pairs.

For the $f_2 \rightarrow \pi^0\pi^0, \eta\eta, \eta\eta'$ transitions, when the f_2 states are mixtures of quarkonium and gluonium components, the rules of quark combinatorics give the following relations for the vertices determined by planar diagrams [8,22]:

$$\begin{aligned} g_{\pi^0\pi^0} &= g \frac{\cos\varphi}{\sqrt{2}} + \frac{G}{\sqrt{2+\lambda}}, \\ g_{\eta\eta} &= g \left(\cos^2\theta \frac{\cos\varphi}{\sqrt{2}} + \sin^2\theta \sqrt{\lambda} \sin\varphi \right) + \frac{G}{\sqrt{2+\lambda}} (\cos^2\theta + \lambda \sin^2\theta), \\ g_{\eta\eta'} &= \sin\theta \cos\theta \left[g \left(\frac{\cos\varphi}{\sqrt{2}} - \sqrt{\lambda} \sin\varphi \right) + \frac{G}{\sqrt{2+\lambda}} (1-\lambda) \right]. \end{aligned} \quad (7)$$

The terms proportional to g stand for the $q\bar{q} \rightarrow \text{two mesons}$ transitions, while the terms with G represent $glueball \rightarrow \text{two mesons}$. Consequently, g^2 and G^2 are proportional to the probabilities for finding quark-antiquark ($1-W$) and glueball (W) components in the considered f_2 -meson: $g^2 = (1-W)g_0^2$ and $G^2 = WG_0^2$, where g_0 and G_0 are couplings for pure states i.e. for transitions of the type presented in Fig. 2.

Let us consider decays into a definite channel, for example $q\bar{q} \rightarrow \pi^0\pi^0$ and $gg \rightarrow \pi^0\pi^0$. If so, the flavour content of the quark loop is fixed, and the rules of the $1/N$ expansion give $g(q\bar{q} \rightarrow \pi^0\pi^0) \sim 1/\sqrt{N_c}$, $g(gg \rightarrow \pi^0\pi^0) \sim 1/N_c$. Normalizing the constants as in (7), this means $G_0^2/g_0^2 \sim 1/N_c$. θ is the mixing angle for the $n\bar{n}$ and $s\bar{s}$ components in the η and η' mesons ($\eta = n\bar{n} \cos\theta - s\bar{s} \sin\theta$ and $\eta' = n\bar{n} \sin\theta + s\bar{s} \cos\theta$); here we neglect the possible admixture of a glueball component to η and η' (according to [23], the glueball admixture to η is less than 5%, to η' — less than 20%). For the mixing angle θ we assume $\theta = 37^\circ$.

Considering a glueball state, we have $\varphi \rightarrow \varphi_{glueball}$; if so, the relations (5) turn into

$$\begin{aligned} g_{\pi^0\pi^0} &\rightarrow g_{\pi^0\pi^0}^{(glueball)} = \frac{g+G}{\sqrt{2+\lambda}}, \\ g_{\eta\eta} &\rightarrow g_{\eta\eta}^{(glueball)} = \frac{g+G}{\sqrt{2+\lambda}} (\cos^2\theta + \lambda \sin^2\theta) \\ g_{\eta\eta'} &\rightarrow g_{\eta\eta'}^{(glueball)} = \frac{g+G}{\sqrt{2+\lambda}} (1-\lambda) \sin\theta \cos\theta. \end{aligned} \quad (8)$$

Hence, in spite of the unknown quarkonium components in the glueball, there are definite relations between the couplings of the glueball state with the channels $\pi^0\pi^0, \eta\eta, \eta\eta'$ which can serve as signatures to define it.

B. Data for relations between coupling constants of f_2 resonance decays into $\pi^0\pi^0, \eta\eta, \eta\eta'$ channels indicating that $f_2(2000)$ is a glueball

The analysis of the reactions $p\bar{p} \rightarrow \pi^0\pi^0, \eta\eta, \eta\eta'$ carried out in [2,17] provides us with the parameters of the tensor resonances $f_2(1920)$, $f_2(2000)$, $f_2(2020)$, $f_2(2240)$, $f_2(2300)$. In Fig. 3, we demonstrate the cross sections for $p\bar{p} \rightarrow \pi^0\pi^0, \eta\eta, \eta\eta'$ in 3P_2 and 3F_2 waves (dashed and dotted lines) and the total ($J=2$) cross section (solid line) as well as Argand-plots for the 3P_2 and 3F_2 wave amplitudes at invariant masses $M = 1.962, 2.050, 2.100, 2.150, 2.200, 2.260, 2.304, 2.360, 2.410$ GeV. The data for $p\bar{p} \rightarrow \pi^+\pi^-$ [24,25] at $1.900 \leq M \leq 2.200$ GeV are also included into analysis.

These amplitudes give us the following ratios $g_{\pi^0\pi^0} : g_{\eta\eta} : g_{\eta\eta'}$ for the f_2 mesons:

$f_2(1920)$	$1 : 0.56 \pm 0.08 : 0.41 \pm 0.07$
$f_2(2000)$	$1 : 0.82 \pm 0.09 : 0.37 \pm 0.22$
$f_2(2020)$	$1 : 0.70 \pm 0.08 : 0.54 \pm 0.18$
$f_2(2240)$	$1 : 0.66 \pm 0.09 : 0.40 \pm 0.14$
$f_2(2300)$	$1 : 0.59 \pm 0.09 : 0.56 \pm 0.17.$

(9)

For the glueball state the relations between the coupling constants are $1 : (\cos^2 \Theta + \lambda \sin^2 \Theta) : (1 - \lambda) \cos \Theta \sin \Theta$. For $(\lambda = 0.5, \Theta = 37^\circ)$ we have $1 : 0.82 : 0.24$, and for $(\lambda = 0.85, \Theta = 37^\circ)$, correspondingly, $1 : 0.95 : 0.07$. Consequently, the relations between the coupling constants $g_{\pi^0 \pi^0} : g_{\eta\eta} : g_{\eta\eta'}$ for the glueball have to be

$$2^{++} \text{ glueball} \quad g_{\pi^0 \pi^0} : g_{\eta\eta} : g_{\eta\eta'} = 1 : (0.82 - 0.95) : (0.24 - 0.07). \quad (10)$$

It follows from the expression (9) that only the coupling constants of the broad $f_2(2000)$ resonance are inside the $0.82 \leq g_{\eta\eta}/g_{\pi^0 \pi^0} \leq 0.95$ and $0.24 \geq g_{\eta\eta'}/g_{\pi^0 \pi^0} \geq 0.07$ intervals. Hence, it is just this resonance which can be considered as a candidate for a tensor glueball, while λ is fixed in the interval $0.5 \leq \lambda \leq 0.7$. Taking into account that there is no place for $f_2(2000)$ on the (n, M^2) -trajectories (see Fig. 1), it becomes clear that indeed, this resonance is the lowest tensor glueball.

C. The analysis of the quarkonium-gluonium contents of $f_2(1920)$, $f_2(2020)$, $f_2(2240)$, $f_2(2300)$

Making use of the data (9), the expression (7) allows us to find φ as a function of the ratio G/g of the coupling constants. The result for the resonances $f_2(1920)$, $f_2(2020)$, $f_2(2240)$, $f_2(2300)$ is shown in Fig. 4. Solid curves enclose the values of $g_{\eta\eta}/g_{\pi^0 \pi^0}$ for $\lambda = 0.6$ (this is the zone $\eta\eta$ in the $(G/g, \varphi)$ plane) and dashed curves enclose $g_{\eta\eta'}/g_{\pi^0 \pi^0}$ for $\lambda = 0.6$ (the zone $\eta\eta'$). The values of G/g and φ , lying in both zones describe the experimental data (9): these regions are shadowed in Fig. 4.

The correlation curves in Fig. 4 enable us to give a qualitative estimate for the change of the angle φ (i.e. the relation of the $n\bar{n}$ and $s\bar{s}$ components in the f_2 meson) depending on the value of the gluonium admixture. The values g^2 and G^2 are proportional to the probabilities of having quarkonium and gluonium components in the f_2 meson, $g^2 = g_0^2(1 - W)$ and $G^2 = G_0^2 W$. Here W is the probability of a gluonium state admixture, and g_0 and G_0 are universal constants. Since $G_0^2/g_0^2 \sim 1/N_c$, we take qualitatively

$$\frac{G^2}{g^2} \simeq \frac{W}{N_c(1 - W)}. \quad (11)$$

Numerical calculations of the diagrams indicate that $1/N_c$ leads to a smallness of the order of $1/10$. Assuming that the gluonium components are less than 20% ($W < 0.2$) in each of the $q\bar{q}$ resonances $f_2(1920)$, $f_2(2020)$, $f_2(2240)$, $f_2(2300)$, we have roughly $W \simeq 10 G^2/g^2$, and obtain for the angles φ the following intervals:

$$\begin{aligned} W_{\text{gluonium}}[f_2(1920)] < 20\% : \quad & -0.8^\circ < \varphi[f_2(1920)] < 3.6^\circ, \\ W_{\text{gluonium}}[f_2(2020)] < 20\% : \quad & -7.5^\circ < \varphi[f_2(2020)] < 13.2^\circ, \\ W_{\text{gluonium}}[f_2(2240)] < 20\% : \quad & -8.3^\circ < \varphi[f_2(2240)] < 17.3^\circ, \\ W_{\text{gluonium}}[f_2(2300)] < 20\% : \quad & -25.6^\circ < \varphi[f_2(2300)] < 9.3^\circ \end{aligned} \quad (12)$$

III. CONCLUSION

Let us summarize what we know about the status of the $(I = 0, J^{PC} = 2^{++})$ mesons in the region of 1900–2400 MeV.

1. The resonances $f_2(1920)$ and $f_2(2120)$ [4] (in [1] they are denoted as $f_2(1910)$ and $f_2(2010)$) are partners in a nonet with $n = 3$ and with a dominant P -component, $3^3 P_2 q\bar{q}$. Ignoring the contribution of the glueball component, their flavour contents, obtained from the reactions $p\bar{p} \rightarrow \pi^0 \pi^0, \eta\eta, \eta\eta'$, are

$$\begin{aligned} f_2(1920) &= \cos \varphi_{n=3} n\bar{n} + \sin \varphi_{n=3} s\bar{s}, \\ f_2(2120) &= -\sin \varphi_{n=3} n\bar{n} + \cos \varphi_{n=3} s\bar{s}, \\ \varphi_{n=3} &= 0 \pm 5^\circ. \end{aligned} \quad (13)$$

2. The next, predominantly 3P_2 states with $n = 4$ are $f_2(2240)$ and $f_2(2410)$ [4]. (By mistake, in [1] the resonance $f_2(2240)$ [2] is listed as $f_2(2300)$, while $f_2(2410)$ [4] is denoted as $f_2(2340)$). Their flavour contents at $W = 0$ are determined as

$$\begin{aligned} f_2(2240) &= \cos \varphi_{n=4} n\bar{n} + \sin \varphi_{n=4} s\bar{s}, \\ f_2(2410) &= -\sin \varphi_{n=4} n\bar{n} + \cos \varphi_{n=4} s\bar{s}, \\ \varphi_{n=4} &= 5 \pm 11^\circ. \end{aligned} \quad (14)$$

3. $f_2(2020)$ and $f_2(2340)$ [4] belong to the basic F -wave nonet ($n = 1$) (in [1] the $f_2(2020)$ resonance [2] is denoted as $f_2(2000)$ and is put in the section "Other light mesons", while $f_2(2340)$ [4] is denoted as $f_2(2300)$). The flavour contents of the 1^3F_2 mesons are

$$\begin{aligned} f_2(2020) &= \cos \varphi_{n(F)=1} n\bar{n} + \sin \varphi_{n(F)=1} s\bar{s}, \\ f_2(2340) &= -\sin \varphi_{n(F)=1} n\bar{n} + \cos \varphi_{n(F)=1} s\bar{s}, \\ \varphi_{n(F)=1} &= 5 \pm 8^\circ. \end{aligned} \quad (15)$$

4. The resonance $f_2(2300)$ has a dominant F -wave component; its flavour content for $W = 0$ is defined as

$$f_2(2300) = \cos \varphi_{n(F)=2} n\bar{n} + \sin \varphi_{n(F)=2} s\bar{s}, \quad \varphi_{n(F)=2} = -8^\circ \pm 12^\circ. \quad (16)$$

A partner of $f_2(2300)$ in the 2^3F_2 nonet has to be a f_2 -resonance with a mass $M \simeq 2570$ MeV. Let us stress once more that there is a resonance, observed in the $\phi\phi$ spectrum, and denoted as $f_2(2300)$ [1]. However, according to the re-analysis [4], its mass was shifted to the region 2340 ± 15 MeV.

5. The broad $f_2(2000)$ state is the lowest tensor glueball. The corresponding coupling constants $f_2(2000) \rightarrow \pi^0\pi^0, \eta\eta, \eta\eta'$ satisfy the relations (8) with $\lambda \simeq 0.5-0.7$. The admixture of the quarkonium component $(q\bar{q})_{glueball}$ in $f_2(2000)$ cannot be determined by the ratios of the coupling constants between the hadronic channels; to define it, $f_2(2000)$ has to be observed in $\gamma\gamma$ -collisions. The value of $(q\bar{q})_{glueball}$ in $f_2(2000)$ may be quite large (of the order of 50%; indeed, the rules of $1/N$ -expansion do not forbid the mixing of gg and $q\bar{q}$). It is, probably, just the largeness of the quark-antiquark component in $f_2(2000)$ which results in its suppressed production in the radiative J/ψ decays [26].

We have now two observed glueballs, a scalar $f_2(1200-1600)$ [12,13] (see also [8,9]) and a tensor $f_2(2000)$ one. Both these glueball states transformed into broad resonances owing to the accumulation of widths of their neighbours. The existence of a low-lying pseudoscalar glueball is also expected. It is natural to assume that it should also turn into a broad resonance. Consequently, the question is, where to look for this broad 0^{-+} state: it can be found both in the region of 1700 MeV, or much higher, ~ 2300 MeV (see the discussion in [26], Section 10.5). In [27] the idea is put forward that the lowest scalar and pseudoscalar glueballs must have roughly equal masses. If so, a 0^{-+} glueball has to occur in the 1700 MeV region.

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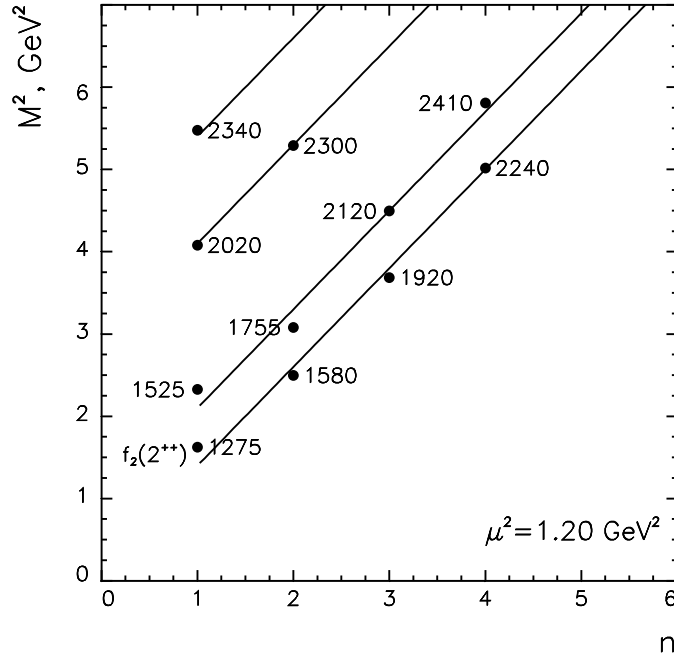


FIG. 1. The f_2 trajectories on the (n, M^2) plane; n is the radial quantum number of the $q\bar{q}$ state. The numbers stand for the experimentally observed f_2 -meson masses M .

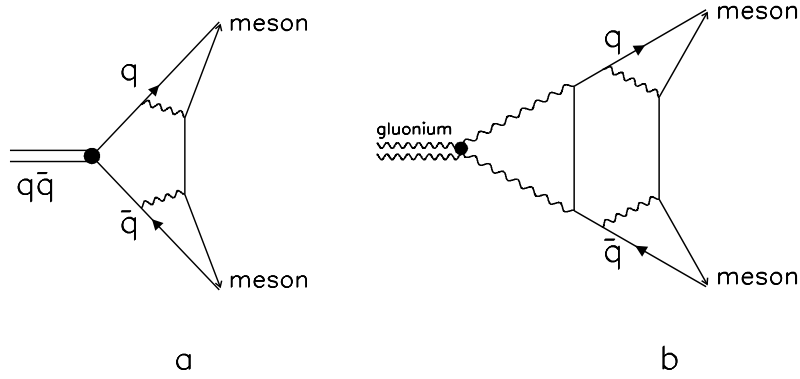


FIG. 2. (a,b) Examples of planar diagrams responsible for the decay of the $q\bar{q}$ -state and the gluonium into two $q\bar{q}$ -mesons (leading terms in the $1/N$ expansion).

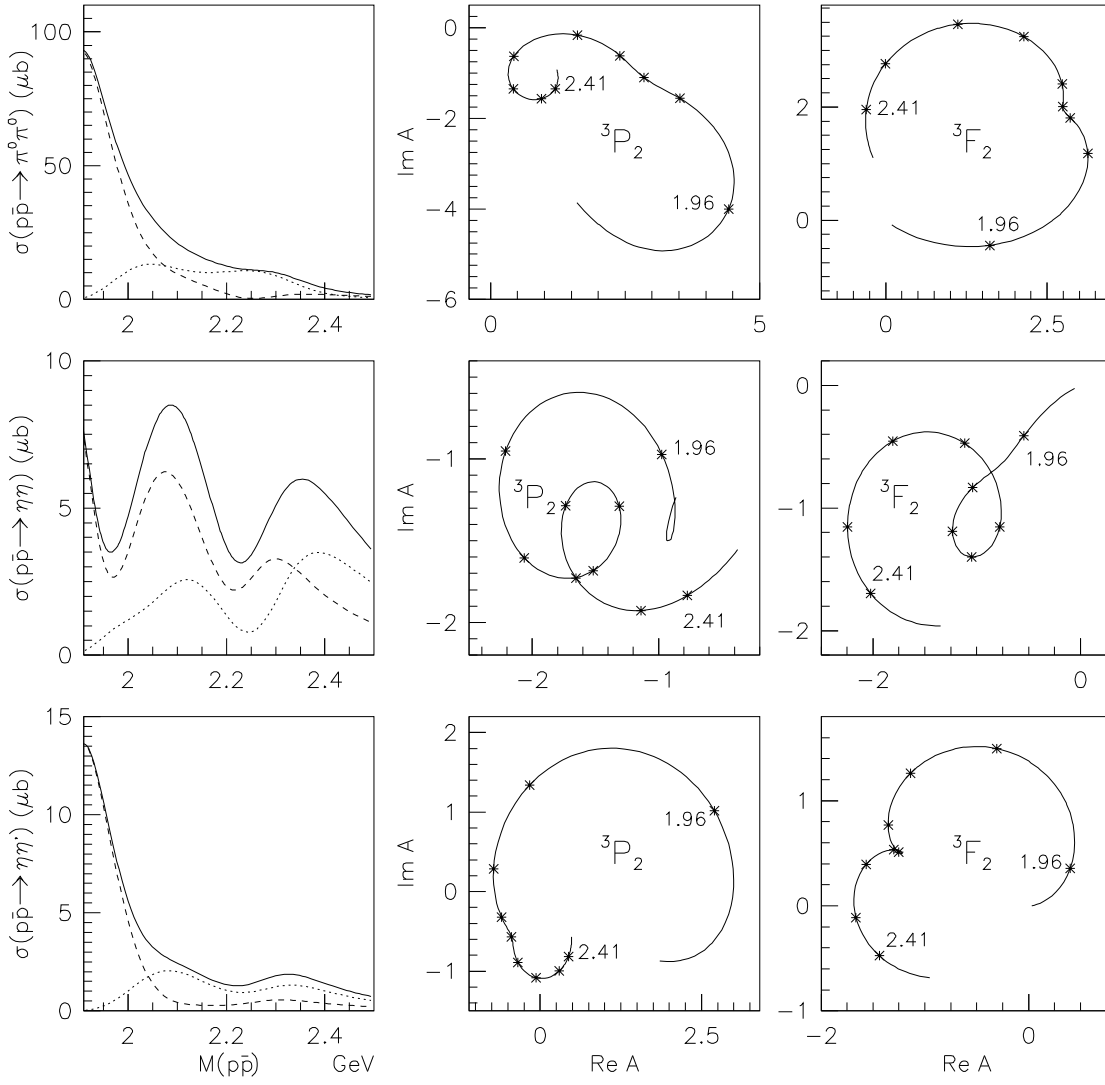


FIG. 3. Cross sections and Argand-plots for 3P_2 and 3F_2 waves in the reaction $p\bar{p} \rightarrow \pi^0\pi^0, \eta\eta, \eta\eta'$. The upper row refers to $p\bar{p} \rightarrow \pi^0\pi^0$: we demonstrate the cross sections for 3P_2 and 3F_2 waves (dashed and dotted lines, correspondingly) and the total ($J=2$) cross section (solid line) as well as Argand-plots for the 3P_2 and 3F_2 wave amplitudes at invariant masses $M = 1.962, 2.050, 2.100, 2.150, 2.200, 2.260, 2.304, 2.360, 2.410$ GeV. The figures on the second and third rows refer to the reactions $p\bar{p} \rightarrow \eta\eta$ and $p\bar{p} \rightarrow \eta\eta'$.

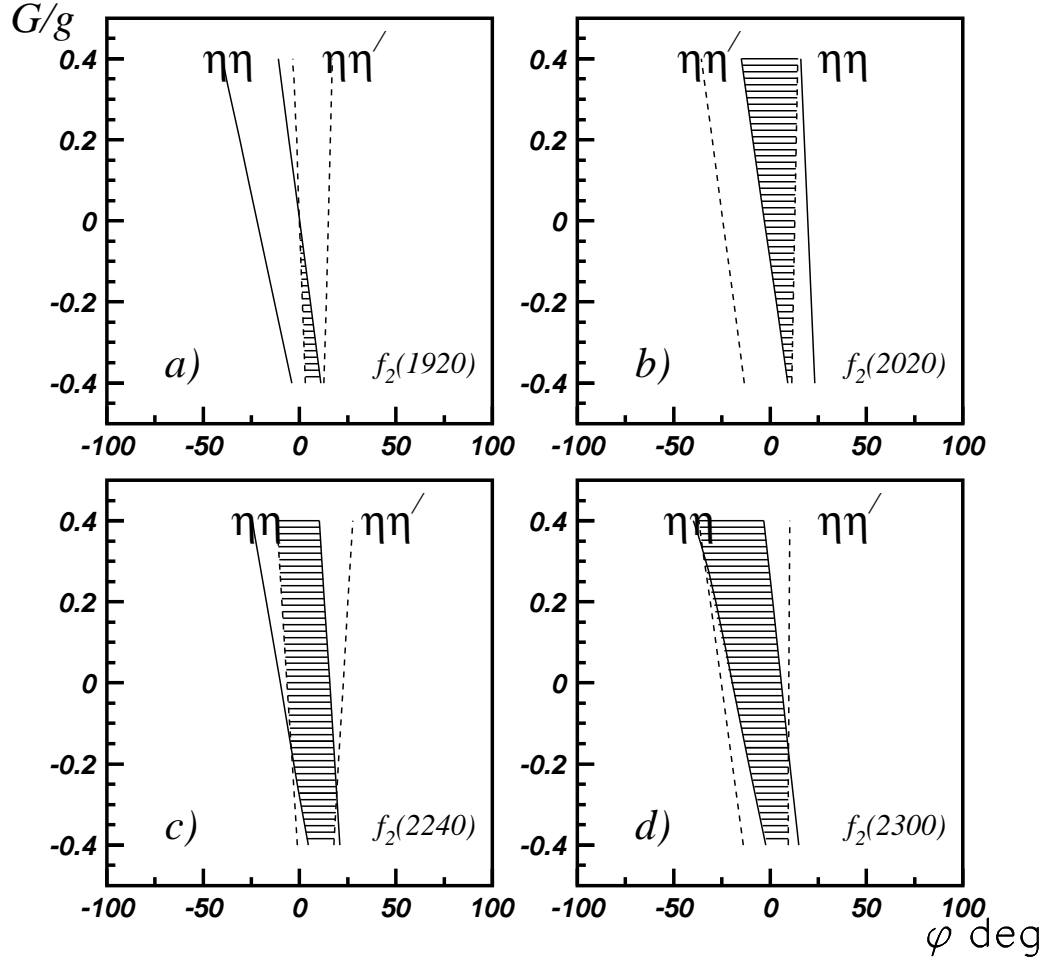


FIG. 4. Correlation curves $g_{\eta\eta}(\varphi, G/g)/g_{\pi^0\pi^0}(\varphi, G/g)$ and $g_{\eta\eta'}(\varphi, G/g)/g_{\pi^0\pi^0}(\varphi, G/g)$ drawn according to (7) at $\lambda = 0.6$ for $f_2(1920)$, $f_2(2020)$, $f_2(2240)$, $f_2(2300)$ [2,17]. Solid and dashed curves enclose the values $g_{\eta\eta}(\varphi, G/g)/g_{\pi^0\pi^0}(\varphi, G/g)$ and $g_{\eta\eta'}(\varphi, G/g)/g_{\pi^0\pi^0}(\varphi, G/g)$ which obey (9) (the zones $\eta\eta$ and $\eta\eta'$ in the $(G/g, \varphi)$ plane). The values of G/g and φ , lying in both zones describe the experimental data (9): these regions are shadowed.